2007 Paper 11 Question 5

Mathematics for Computation Theory

(*a*) Let M be an n-state deterministic finite automaton over the finite alphabet S. Write $l(w)$ for the length of words $w \in S^*$. Suppose that M accepts the word $x \in S^*$, where $l(x) \geqslant n$.

Show that x is a concatenation of words uvw, where $l(uv) \leqslant n$, $l(v) \geqslant 1$, and M accepts the word $z_k = u\, v^k\, w$ for all natural numbers $k \geqslant 0$. [10 marks]

(*b*) Let $S = \{a, b\}$ be an alphabet of two symbols. Explain with proof whether each of the following languages over S is regular, giving a regular expression denoting the language if so:

(*i*) the set of words $w \in S^*$ in which there are more occurrences of b than there are occurrences of a; [5 marks]

(*ii*) the set of words $w \in S^*$ in which each occurrence of a is followed immediately by an occurrence of b. [5 marks]