2007 Paper 10 Question 10

Data Structures and Algorithms

(*a*) Give a clear description of an efficient algorithm for finding the i^{th} smallest element of an n-element vector. Write some pseudocode for the algorithm and discuss its time complexity. Compare it with other plausible ways of achieving the same result. [Notes: Use zero-based indexing. You may assume for simplicity that all the elements of the vector are different.] [4 marks]

(*b*) Give a clear description of an efficient algorithm for finding the k smallest elements of a very large n-element vector. Compare its running time with that of other plausible ways of achieving the same result, including that of applying k times your solution for part (*a*). [Note that in part (*a*) the result of the function consists of one element, whereas here it consists of k elements. As above, you may assume for simplicity that all the elements of the vector are different.] [6 marks]

(*c*) Give an optimal algorithm for solving part (*b*) for $k = 1$. Give the worst-case number of comparisons performed by your algorithm as a function of n. [Note: exact *number of comparisons*, not just asymptotic complexity.] [4 marks]

(*d*) Same as part (*c*), but for $k = 2$. [6 marks]