2006 Paper 8 Question 16

Information Theory and Coding

(*a*) Suppose we know the conditional entropy $H(X|Y)$ for two slightly correlated discrete random variables X and Y. We wish to guess the value of X, from knowledge of Y. There are $\mathcal{N}$ possible values of X. Give a lower bound estimate for the probability of error, when guessing X from knowledge of Y. What is the name of this relationship? [4 marks]

(*b*) In an error-correcting (7/4) Hamming code, under what circumstance is there still a residual error rate? (In other words, what event causes this error-correction scheme to fail?) [2 marks]

(*c*) Broadband noise whose power spectrum is flat is "white noise". If the average power level of a white noise source is σ^2 and its excursions are zero-centred so its mean value is $\mu = 0$, give an expression describing the probability density function $p(x)$ for excursions x of this noise around its mean, in terms of σ. What is the special relationship between the entropy of a white noise source, and its power level σ^2? [4 marks]

(*d*) Explain the phenomenon of aliasing when a continuous signal whose total bandwidth extends to $\pm W$ is sampled at a rate of $f_s < 2W$. If it is not possible to increase the sampling rate f_s, what can be done to the signal before sampling it that would prevent aliasing? [5 marks]

(*e*) Prove that the sinc function,

$$\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

is invariant under convolution with itself: in other words that the convolution of a sinc function with itself is just another sinc function. You might find it useful to recall that the Fourier transform of a sinc function is the rectangular pulse function:

$$\Pi(k) = \begin{cases} \frac{1}{2\pi} & |k| \leq \pi \\ 0 & |k| > \pi \end{cases}$$

[5 marks]