

2006 Paper 4 Question 3

Data Structures and Algorithms

- (a) Define the minimum spanning tree (MST) of a graph and justify, with counterexamples where appropriate, why the search for it makes sense only on connected, weighted and undirected graphs. [2 marks]
- (b) Define these MST expressions: *safe edge*, *cut*, *respecting a set of edges*. [3 marks]
- (c) Describe an efficient MST-finding algorithm, write some clear pseudocode for it and prove its correctness. [9 marks]
- (d) Say whether *each* of the following two statements is true or false, justifying each answer with a proof or a counterexample.
- (i) Graph G has a unique MST \Rightarrow For every cut of G , the lightest edge that crosses it is unique.
- (ii) Graph G has a unique MST \Leftarrow For every cut of G , the lightest edge that crosses it is unique.
- [6 marks]