2006 Paper 3 Question 10

Mathematical Methods for Computer Science

(a) Suppose that $X_1, X_2, \ldots$ is a sequence of random variables. State the Central Limit Theorem, noting any assumptions that you make about the random variables, X_i, and stating carefully the type of limiting convergence that is being considered. [2 marks]

(b) Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from a population with unknown mean μ.

(i) Explain what is meant by a $100(1-\alpha)$ percent confidence interval for the unknown parameter μ. [2 marks]

(ii) Derive an approximate $100(1-\alpha)$ percent confidence interval for μ using the Central Limit Theorem. [4 marks]

(iii) Apply your method to construct an approximate 95% confidence interval for the population mean, μ, given the following random sample of 20 values taken from the population: 10.583, 6.775, 12.126, 9.135, 12.690, 10.638, 11.683, 10.428, 11.201, 7.911, 10.608, 9.477, 9.643, 8.263, 13.909, 8.563, 10.910, 9.962, 14.108, 11.201. [4 marks]

(c) Lift A has a sign reading "`13 people, 1000 kg`" and lift B has a similar sign reading "`4500lb or 30 persons`". Given a standard deviation of $\sigma = 20$ `kg` for the weight of a person, what is an approximate mean weight of a person in `kg` such that the probability a group of maximum size exceeds the weight limit of a lift with probability p? Apply your method in the cases of lifts A and B separately and with the values of $p = 0.1$ and $p = 0.01$. You may use the approximation that 1 `kg` $\approx$ 2.2 `lb`. [8 marks]

The following table gives values of $\Phi(x) = P(Z \leq x)$, the distribution function of Z, where Z is a random variable with the standard normal distribution.

x	0.842	1.282	1.645	1.96	2.326
$\Phi(x)$	0.8	0.9	0.95	0.975	0.99