

2006 Paper 2 Question 6

Discrete Mathematics II

(a) (i) Draw the truth tables to illustrate the truth values of $A \Rightarrow B$ and $A \Leftrightarrow B$ in terms of the truth values of A and B . [2 marks]

(ii) By considering their truth tables, establish the following equivalences of boolean propositions:

1. $A \Leftrightarrow (B \Leftrightarrow C) = (A \Leftrightarrow B) \Leftrightarrow C$. [5 marks]

2. $(F \Leftrightarrow B) = \neg B$, where F is the proposition “false”. [2 marks]

3. $\neg(B \Leftrightarrow C) = ((\neg B) \Leftrightarrow C)$. [2 marks]

(iii) By assigning suitable truth values to propositions B and C , explain why the equivalence 3 above fails to hold if “ \Leftrightarrow ” is replaced by “ \Rightarrow ”. [3 marks]

(b) The set S is defined to be the least subset of (positive) natural numbers \mathbb{N} such that:

$$1 \in S;$$

$$\text{if } n \in S, \text{ then } 3n \in S;$$

$$\text{if } n \in S \text{ and } n > 2, \text{ then } (n - 2) \in S.$$

Show that $S = \{m \in \mathbb{N} \mid \exists r, s \in \mathbb{N} \cup \{0\}. m = 3^r - 2s\}$. [6 marks]