2006 Paper 10 Question 9

Mathematics for Computation Theory

(a) Let A, B C be sets. Define:

(i) the *Cartesian product* $(A \times B)$;

(ii) the set of relations R between A and B;

(iii) the identity relation Δ_A on the set A.

[3 marks]

(b) Suppose S, T are relations between A and B, and between B and C, respectively. Define the inverse relation S^{-1} and the product relation $S \circ T$.

[2 marks]

(c) Let f be a relation between A and B. Characterise the following conditions in terms of the algebra of relations:

(i) f is a partial function;

(ii) f is a total function;

(iii) (total) function f is a surjection (ONTO);

(iv) (total) function f is an injection $(1 - 1)$.

[4 marks]

(d) A total function that is both a surjection and an injection is called a *bijection*. Show that if f is a bijection between A and B, f^{-1} is also a bijection.

[2 marks]

(e) Consider the set of natural numbers $\mathbb{N} = \{0, 1, 2, \ldots\}$. Define relation $f = \{((x, y), z) \mid z = 2^x(2y + 1)\} \subseteq ((\mathbb{N} \times \mathbb{N}) \times \mathbb{N})$. Which of conditions ($i$)–($iv$) in part ($c$) does relation f between $(\mathbb{N} \times \mathbb{N})$ and $\mathbb{N}$ satisfy? [6 marks]

(f) Show how to modify f to establish a bijection $h : \mathbb{N} \to (\mathbb{N} \times \mathbb{N})$. [3 marks]