2005 Paper 8 Question 16

Computer Systems Modelling

Let X be a random variable taking values in the discrete state space $\{1, 2, \ldots, 6\}$ representing the outcome of a fair die with distribution

$$P(X = i) = \frac{1}{6} \qquad i = 1, 2, \ldots, 6\,.$$

(a) Suppose that you have been given a function `r()` that claims to return pseudo-random numbers with the distribution $U(0, 1)$. Suppose also that you have used this function to generate an independent sample of size 150 of values of X with outcomes given in the following table. Given the relatively high frequency of the outcome $i = 6$ in your sample you may be concerned that the function `r()` is biased. Explain how a goodness of fit test can be used to test for such a bias.

face, i	1	2	3	4	5	6
number of outcomes	22	21	22	27	22	36

[4 marks]

(b) Describe how to apply the χ^2 (Chi-squared) goodness of fit test to your sample.

The following table gives values of t such that if T is a χ^2 random variable with d degrees of freedom then $P(T > t) = 0.05$. Do you conclude from this test that the function `r()` is biased or not at the 5% level?

degrees of freedom d	1	2	3	4	5	6
t	3.84	5.99	7.81	9.49	11.07	12.59

[6 marks]

(c) Suppose that you have implemented a discrete event simulator for a FIFO M/G/1 queueing model for the processing of tasks in a computer system. Explain what probabilistic modelling assumptions are made in specifying such a simulator. Given a log of events and event times generated by your simulator explain what tests you would use to validate your simulation approach.

[10 marks]