2005 Paper 6 Question 12

Complexity Theory

(a) When defining space-complexity we use two-tape Turing Machines, with a read-only tape for input data and a read–write working tape. We count only the space used on the work tape. What difference would be made if we worked with standard one-tape Turing machines, loaded the input data onto the tape to start with and measured space in terms of total tape cells touched by the end of the computation? [4 marks]

(b) Comment on the following: "The problem of finding a factor of a number N is NP, because if we have a factor P of N we can do a simple trial division and check it in time related to $\log(N)$. Thus finding factors of numbers of the form $2^p - 1$ (these are known as Mersenne Numbers) is a problem in the class NP". [2 marks]

(c) Define the class co-NP. State an example of a problem that lies in it. [2 marks]

(d) What is a *witness function* for an NP problem? Why might some problem such as 3-SAT have many different witness functions associated with it? [2 marks]

(e) Give and justify a relation between $NTIME(f(n))$ and $SPACE(f(n))$. [4 marks]

(f) Matchings on bi-partite graphs can be found in polynomial time. The matching problem on tri-partite graphs is known to be NP-complete. Does this suggest that the corresponding problem with a graph whose nodes are partitioned into four sets ("quad-partite" matching) is liable to be exponential in complexity? Justify your answer. [4 marks]

(g) Comment on the following proposition: "Determining which player can force a win from a given starting position in the game of Chess is an NP problem because given any sequence of moves it will be easy to verify that they are all legal moves and easy to see who wins at the end of them." [2 marks]