2005 Paper 6 Question 1

Data Structures and Algorithms

A certain program has to maintain an array, `count`, of N counters which are all initialised to zero. The value of counter i can be incremented by one by the call: `increment(i)`, and this is the only way the program changes counter values. Two variables, `mincount` and `maxcount`, must always hold the smallest and largest of the counter values whenever the point of execution is not within the function `increment`. You may assume that `increment` is called about $1000N$ times when the program is run and that its argument is typically uniformly randomly distributed between 1 and N, but on some runs it cycles through the numbers 1 to N in order 1000 times.

(a) Describe, in detail, an efficient data structure and algorithm to use when N is expected to be about 10. [5 marks]

(b) Describe, in detail, an alternative data structure and algorithm to use when N is about a million. [10 marks]

(c) Suppose your algorithm for (a) above were used when $N = 10^6$, estimate how much slower it would be compared with your algorithm for (b). [5 marks]