

## 2005 Paper 11 Question 8

### Numerical Analysis I

- (a) Define *absolute error* and *relative error*. How are these related? Explain briefly the term *loss of significance*. [3 marks]
- (b) An algorithm is required for solution of  $ax^2 + bx + c = 0$  where  $b > 0$ . Describe how loss of significance can occur in the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

and derive an alternative formula for such a case. Illustrate your answer by applying it to the case  $a = 30$ ,  $b = 3000$ ,  $c = 1$  on a decimal machine with only 5 significant digits available. [You should assume for the purposes of calculation that  $\sqrt{b^2 - 4ac}$  evaluates to  $b - (2ac/b)$ , correctly rounded.] [10 marks]

- (c) The series

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

is to be summed by taking terms in order, left to right, using only  $p$  decimal digits of precision until additional terms are negligible. If  $x = 6$  find the largest term of the series and hence, assuming  $\cos 6 \simeq 1$ , estimate roughly how many decimal digits of accuracy will be lost in the process. [7 marks]