

## 2004 Paper 2 Question 4

### Probability

A computer room is cooled by two groups of fans. Each group contains three fans. All six fans are identical and, when they fail, they fail equiprobably and independently. Two or more fans never fail simultaneously.

There is deliberate over-provision of fans so that there is adequate cooling as long as at least one fan in each group is working. The local maintenance policy is to do nothing until all three fans in one or other of the groups have failed. When that happens, all six fans in the room are replaced including those that have not failed.

- (a) When replacement occurs, what is the minimum number of *working fans* that could be replaced and what is the maximum number that could be replaced? Justify your answers. [2 marks]
- (b) Let  $X$  be a random variable whose value  $r$  is the number of *failed fans* when replacement occurs. Clearly  $0 \leq r \leq 6$  and, for some values of  $r$ , the probability  $P(X = r) = 0$ . By constructing an event tree or otherwise, tabulate  $P(X = r)$  for  $r = 0, 1, 2, \dots, 6$ . All non-zero probabilities should be expressed as fractions. [12 marks]
- (c) Determine the expectation and variance  $E(X)$  and  $V(X)$ . [6 marks]