2003 Paper 9 Question 10

Types

(*a*) Describe the relation $=_\beta$ of *beta-conversion* between terms of the polymorphic lambda calculus (PLC). How can one decide whether two typeable PLC terms are in this relation? Why does the decision procedure fail for untypeable terms? [8 marks]

(*b*) Let ω be the polymorphic type $\forall\alpha_1((\forall\alpha_2(\alpha_2 \to \alpha_1)) \to \alpha_1)$. Show that there is a closed PLC term I with the following two properties.

(*i*) I has type $\forall\alpha(\alpha \to \omega)$.

(*ii*) If M_1 and M_2 are any closed PLC terms of the same type, τ say, and if $(I\,\tau\,M_1) =_\beta (I\,\tau\,M_2)$, then $M_1 =_\beta M_2$.

[Hint: for property (*ii*), consider the beta-normal forms of $I\,\tau\,M_1\,\alpha\,x$ and $I\,\tau\,M_2\,\alpha\,x$, where α is a type variable and x is a variable.] [12 marks]