

2003 Paper 8 Question 15

Computer Systems Modelling

(a) Describe the congruential class of methods for generating pseudo-random numbers from a uniform $(0,1)$ distribution. [3 marks]

(b) Let U be a uniform $(0,1)$ random variable. Show that for any continuous probability distribution function $F(x)$ the random variable, X , defined by

$$X = F^{-1}(U)$$

has the probability distribution function $F(x)$. [3 marks]

(c) Suppose that X_1, X_2, \dots, X_n are independent identically distributed random variables with mean μ and variance σ^2 . Define the sample mean, \bar{X} , and sample variance, S^2 . [2 marks]

(d) Calculate the expectation and variance of \bar{X} . [4 marks]

(e) Use the central limit theorem to derive an approximate $100(1 - \alpha)$ percent confidence interval for estimating μ . [6 marks]

(f) Describe an algorithm to control the length of a simulation run such that the estimate obtained has an approximate $100(1 - \alpha)$ confidence interval of length at most ℓ . [2 marks]