

2003 Paper 8 Question 10

Information Theory and Coding

(a) An alphabet has six symbols with the following probabilities of occurrence:

a	b	c	d	e	f
0.31	0.18	0.21	0.08	0.17	0.05

(i) Construct an optimal prefix code (an Huffman code) for this alphabet. [5 marks]

(ii) What is the average number of bits per symbol in your code? Is this greater than, less than, or equal to the entropy of the alphabet? [3 marks]

(b) Jane's Furniture Store sells 2-, 3- and 4-seater sofas in two styles: classic and modern. At the end of May, Jane tallies the number of each type that has been sold in each style during May.

Style	Type (no. of seats)		
	2	3	4
Classic	8	24	4
Modern	8	8	12

(i) Let X be the type of sofa (2-, 3-, or 4-seats) and Y be the style (classic or modern). Calculate the values of $H(X)$, $H(Y)$, $H(X, Y)$, $H(X|Y)$, $H(Y|X)$, and $I(X; Y)$. [9 marks]

(ii) In part (b)(i), you should have found that $I(X; Y) \neq 0$. For the month of June, Jane wants to have $I(X; Y) = 0$ while still selling the same number of sofas of each type as in May. Change the numbers in the table so that the same number of sofas of each type are sold (i.e. the column totals remain unchanged) but so that $I(X; Y) = 0$. What is $H(X, Y)$ in this revised table? [3 marks]