

2003 Paper 7 Question 5

Computer Systems Modelling

Let $N(t)$ denote the number of events in the time interval $[0, t]$ for a (homogeneous) Poisson process of rate λ , ($\lambda > 0$).

- (a) State the necessary properties on $N(t)$ that define a (homogeneous) Poisson process of rate λ . [4 marks]
- (b) By dividing the interval $[0, t]$ into equal length sub-intervals show that $N(t)$ is a Poisson random variable with mean λt . [4 marks]
- (c) Let X_1 denote the time of the first event and for $n > 1$ let X_n denote the elapsed time between the $(n - 1)$ th and the n th events of the Poisson process. Determine the distribution of X_1 and the joint distribution of X_1 and X_2 . [4 marks]
- (d) Let $S_n = \sum_{i=1}^n X_i$ denote the time of the n th event. Derive the probability density function of the random variable $S_n(t)$. [4 marks]
- (e) Give an algorithm to generate the first T time units of a (homogeneous) Poisson process of rate λ . [4 marks]