

## 2003 Paper 6 Question 12

### Semantics of Programming Languages

Consider the language below, in which  $r$  ranges over the reals  $\mathbb{R}$  and  $n$  ranges over the subset  $\mathbb{Z} \subset \mathbb{R}$  of integers.

$$T ::= \text{int} \mid \text{real} \mid \text{bool} \mid T \rightarrow T \mid \{lab : T, \dots, lab : T\}$$

$$e ::= n \mid r \mid \mathbf{true} \mid \mathbf{false} \mid \mathbf{fn} \ x : T \Rightarrow e \mid e \ e \mid x \mid \{lab = e, \dots, lab = e\} \mid \#lab \ e$$

To allow any `int` to be used as a `real` we can use a subtype relation that satisfies

$$(\text{num}) \frac{}{\text{int} <: \text{real}}$$

Suppose the typing rules are the standard ones, including the subsumption rule:

$$(\text{sub}) \frac{\Gamma \vdash e : T \quad T <: T'}{\Gamma \vdash e : T'}$$

(a) Give the remainder of the rules required to define the subtype relation  $T <: T'$ . [7 marks]

(b) For each of the following expressions, either give a type derivation (including derivations of any subtype relationships used) or explain why it is not typeable.

(i)  $(\mathbf{fn} \ x : \{p : \text{real}\} \Rightarrow \#p \ x)\{p = 1, q = \mathbf{true}\}$

(ii)  $(\mathbf{fn} \ x : \{q : \text{bool}\} \Rightarrow \#p \ x)\{p = 2.7, q = \mathbf{true}\}$

(iii)  $(\mathbf{fn} \ x : \{r : \{p : \text{int}, q : \text{bool}\}\} \Rightarrow \#p(\#r \ x))\{r = \{p = 3\}\}$

[11 marks]

(c) List all the types  $T$  for which  $\{\} \vdash (\mathbf{fn} \ y : \text{real} \Rightarrow 6) : T$  is derivable. [2 marks]