

2003 Paper 5 Question 9

Logic and Proof

(a) For $k > 0$, let $\phi(k)$ be the formula

$$[(P_1 \leftrightarrow Q_1) \wedge \dots \wedge (P_k \leftrightarrow Q_k)] \rightarrow R.$$

Prove that there exists an ordering of the propositional variables, namely $P_1, P_2, \dots, Q_1, Q_2, \dots, R$, such that the size of the ordered binary decision diagram (OBDD) for $\phi(k)$ increases linearly with k . [5 marks]

(b) Prove that there exists a variable ordering such that the size of the OBDD for $\phi(k)$ increases exponentially with k . [6 marks]

(c) Give a set of clauses suitable for attempting to prove $\phi(k)$ using resolution. [3 marks]

(d) Describe the computation that would result if the Davis–Putnam (DPLL) procedure were applied to these clauses. [6 marks]