

## 2003 Paper 5 Question 10

### Foundations of Functional Programming

Suppose that lists are to be represented in a pure functional manner using a convention where, for instance, a list with three members  $a_1$ ,  $a_2$  and  $a_3$  is modelled by a lambda term

$$\lambda f.\lambda x.f a_1(f a_2(f a_3 x))$$

- (a) Give the lambda term that corresponds to an empty list. [2 marks]
- (b) Explain how the normal list operations can be achieved on lists that are represented in this way. Specifically show how to create pure functional implementations of
- (i) a test for an empty list, [2 marks]
  - (ii) adding a new item to the front of a list, and [4 marks]
  - (iii) finding the head and tail of a non-empty list. [9 marks]
- (c) Show how a map function can be implemented for use with these functional lists, so that

`map f [a,b,c] -> [f a, f b, f c]`

[3 marks]