

2003 Paper 4 Question 9

Computation Theory

What is the *Church–Turing Thesis*? Briefly describe some evidence that it is true. [4 marks]

Using the Church–Turing Thesis, or otherwise, show that if $f(x)$ and $g(x)$ are partial recursive functions of a single argument, then so are the following functions, where $\text{dom}(f)$ denotes the set of integers x for which $f(x)$ is defined, and similarly for $\text{dom}(g)$.

$$h(x) = \begin{cases} x & \text{if } x \in \text{dom}(f) \text{ and } x \in \text{dom}(g) \\ \text{undefined} & \text{otherwise} \end{cases} \quad [4 \text{ marks}]$$

$$k(x) = \begin{cases} x & \text{if } x \in \text{dom}(f) \text{ or } x \in \text{dom}(g) \\ \text{undefined} & \text{otherwise} \end{cases} \quad [6 \text{ marks}]$$

Is the partial function defined by

$$f'(x) = \begin{cases} x & \text{if } x \notin \text{dom}(f) \\ \text{undefined} & \text{otherwise} \end{cases}$$

necessarily partial recursive if f is? Justify your answer. [6 marks]