2003 Paper 12 Question 12

Complexity Theory

If $A \subseteq \Sigma_1^*$ and $B \subseteq \Sigma_2^*$ are two languages over the alphabets Σ_1 and Σ_2 respectively, we write $A \leq_P B$ to denote that A is polynomial-time reducible to B.

(a) Give a precise definition of $\leq_P$ [2 marks]

(b) Is the relation $\leq_P$ on languages:

(i) reflexive?

(ii) symmetric?

(iii) transitive?

Give a proof for your answer in each case. [9 marks]

(c) If Σ is an alphabet, show that if P = NP then every language $L \subseteq \Sigma^*$ in NP is NP-complete except $\emptyset$ and Σ^*. Why are these two exceptions not NP-complete? [9 marks]