COMPUTER SCIENCE TRIPOS Part IA

Tuesday 3 June 2003 1.30 to 4.30

Paper 2

Answer the question in Section A, **one** *question from each of Sections B and C, and* **two** *questions from Section D.*

Submit the answers in five **separate** *bundles, each with its own cover sheet. On each cover sheet, write the numbers of* **all** *attempted questions, and circle the number of the question attached.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 Multi-part question

Answer all parts.

(*a*) (*i*) Provide at least *one* argument in favour of computer cracking. [1 mark]

(*ii*) Name at least *two* of the three new offences created by the *Computer Misuse Act 1990*. [2 marks]

(*iii*) State at least *one* of the two extensions of this basic law. [1 mark]

(*b*) What are the five stages of the Capability Maturity Model? [4 marks]

(*c*) In the context of loop design, what is a *sentinel*? Illustrate your answer by coding a loop that finds the first non-negative number in an array. [4 marks]

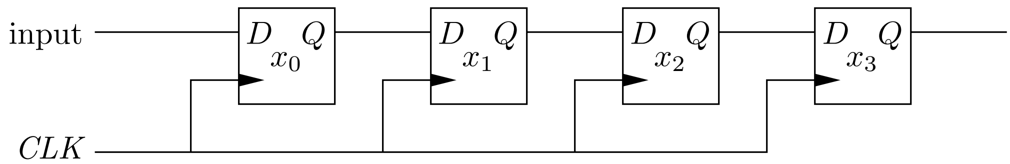
(*d*) If a deterministic finite automaton accepts any string at all, why does it accept one whose length is less than the number of states in the automaton? [4 marks]

(*e*) Give a logic symbol for a dual-port register file containing eight four-bit registers. Be sure to name each input and output connection and give its width. What non-determinism can arise? [4 marks]

SECTION B

2 Digital Electronics

(*a*) A 4-bit shift register constructed from edge-triggered D-type flip flops is shown below. If, on successive rising edges of the clock signal *CLK*, the input takes on the values $1, 0, 1, 0, 1, 1, 1, 0$, what are the contents of the shift register after each edge of the clock? You may assume that the register contains all zeroes initially.



[4 marks]

(*b*) Using a (possibly larger) shift register, show how one may detect a particular pattern in the input shown. As an example, use the 8-bit pattern `0xF0`. High-order bits precede low-order bits in the input stream. [4 marks]

(*c*) The input stream is framed by a one byte frame pattern (`0xF0`) every 256 bytes. However, the frame pattern may also appear at an arbitrary position in the input stream.

It is required to design a framing circuit which generates two outputs: *framelock*, which is asserted when the circuit "believes" it has determined where the frame boundaries are, and *frame pointer*, which is asserted on the clock edge immediately after the frame marker is detected. The circuit "believes" itself to be locked to the frame structure when two successive frame patterns have been found 256 bytes (i.e. 2048 bits) apart. The circuit should not respond to unaligned frame patterns while it believes itself to be in lock or if, once in lock, it has missed fewer than two expected frame patterns.

Draw a state diagram for the finite state control of the circuit. You may assume the existence of an 11-bit resettable counter. You should consider the process of assuming lock, maintaining lock and the accommodation of a single missed framing pattern. State explicitly any additional assumptions you make.

[10 marks]

(*d*) Outline the complexity in gates and flip flops for an implementation of the framing circuit. [2 marks]

3 Digital Electronics

(*a*) Design a 2-bit multiplier for unsigned integers which takes input $x_1\,x_0$ representing the unsigned integer X, $y_1\,y_0$ representing the unsigned integer Y, and produces the output $z_3\,z_2\,z_1\,z_0$ representing the unsigned integer Z. [4 marks]

(*b*) How can multipliers designed in part (*a*) be cascaded (with adders) to provide a four-bit multiplier? [4 marks]

(*c*) Design a sequential 8-bit multiplier. You can assume that a 16-bit adder has been provided. The finite state control can be described by a state diagram. [8 marks]

(*d*) Outline the design of a sequential divider which can divide 16-bit unsigned integers by 8-bit unsigned integers. [4 marks]

SECTION C

4 Probability

In order to test the integrity of a network of ducting, engineers have developed an inspection device which can be introduced at a node and which then finds its way along a length of ducting to an adjacent node.

In a particular case, eight nodes are sited at the vertices (corners) of a cube and 12 lengths of ducting are arranged along the edges of the cube.

The inspection device is introduced at one node and equiprobably chooses one of the three lengths of ducting leading from that node for its first move. On arrival at the adjacent node the device equiprobably chooses one of the three lengths of ducting leading from that node (including the length it has just inspected). It continues in this fashion until the engineers stop its operation.

Let A_n be the probability of the inspection device returning to the starting node after n moves, and deem $A_0 = 1$.

Let D_n be the probability of the inspection device visiting the node diagonally opposite the starting node after n moves. Clearly, $D_0 = 0$.

(*a*) Demonstrate that $A_n = 0$ for all odd n and that $D_n = 0$ for all even n. [4 marks]

(*b*) Determine A_2, A_4 and A_6 expressing all values as fractions. [8 marks]

(*c*) To what value does A_n tend as (even) n increases indefinitely? [4 marks]

(*d*) By noting a pattern in the values of A_2, A_4 and A_6 or otherwise, give (without proof) an expression for the value of A_n for arbitrary n. [4 marks]

5 Probability

(*a*) If a continuous probability density function (p.d.f.) $f(x)$ is transformed by some transformation function $y(x)$ into a new p.d.f. $g(y)$, then:

$$g(y) = f\big(x(y)\big)\left|\frac{dx}{dy}\right|$$

What constraints are there on the function $y(x)$ and its inverse $x(y)$? What is the significance of the vertical bars round $\frac{dx}{dy}$? [4 marks]

(*b*) Suppose that X is a continuous random variable distributed Uniform(0,1). Its p.d.f. $f(x)$ is given by:

$$f(x) = \begin{cases} 1, & \text{if } 0 \leqslant x < 1 \\ 0, & \text{otherwise} \end{cases}$$

What four transformation functions are required to transform $f(x)$ into the following:

(*i*)

$$g(y) = \begin{cases} \lambda.e^{-\lambda y}, & \text{if } y > 0 \\ 0, & \text{otherwise} \end{cases}$$

[4 marks]

(*ii*)

$$g(y) = \begin{cases} \sin y, & \text{if } 0 \leqslant y < \frac{\pi}{2} \\ 0, & \text{otherwise} \end{cases}$$

[4 marks]

(*iii*)

$$g(y) = \begin{cases} \frac{1}{2}(2-y), & \text{if } 0 \leqslant y < 2 \\ 0, & \text{otherwise} \end{cases}$$

[4 marks]

(*iv*)

$$g(y) = \begin{cases} \frac{3}{8}(2-y)^2, & \text{if } 0 \leqslant y < 2 \\ 0, & \text{otherwise} \end{cases}$$

[4 marks]

SECTION D

6 Computer Perspectives

You start using a new computer, which has an implementation of ML. You write a new ML program which is intended to produce a result of type `int`, i.e. an integer. You run it, and see on your screen (which we assume works properly) the following response:

```
1.99 :  int
```

Discuss the following as possible reasons for the response:

(*a*) your program is faulty;

(*b*) the computer's calculation with real numbers is inaccurate;

(*c*) the ML implementation is faulty;

(*d*) ML is a badly designed language.

As part of your answer, discuss the assertion that "a type system is a way of ensuring that every program does what it was supposed to do".

[20 marks]

7 Software Engineering I

Software engineering academics said for years that a significant percentage of large software projects failed. In the run-up to Y2K, most of the world's large companies claimed that fixing the Millennium Bug was a large project whose success was critical to their survival. One would therefore expect many large companies to have failed, but none did. Who was mistaken? Justify your answer. [20 marks]

8 Software Engineering II

(*a*) Present the top-down design of a program that generates calendars for a given month and year, as in this example:

```
    October 2003
 S  M Tu  W Th  F  S
           1  2  3  4
 5  6  7  8  9 10 11
12 13 14 15 16 17 18
19 20 21 22 23 24 25
26 27 28 29 30 31
```

Express your program using a readable pseudo-code, carefully outlining the program's design. You may assume that primitives for weekday and date calculations are provided. [10 marks]

(*b*) Consider the following ML function declarations:

```
fun app([], ys)     = ys
  | app(x::xs, ys) = x :: app(xs,ys);
fun nlength []      = 0
  | nlength (x::xs) = nlength xs + 1;
fun nrev  []    = []
  | nrev(x::xs) = app (nrev xs, [x]);
```

Use structural induction (on `xs`) to prove the equations

(*i*) `nlength(app(xs,ys)) = nlength xs + nlength ys`, and

(*ii*) `nlength(nrev xs) = nlength xs`.

[10 marks]

9 Regular Languages and Finite Automata

(*a*) Let L be the set of all strings over the alphabet $\{a, b\}$ that end in a and do not contain the substring bb. Describe a deterministic finite automaton whose language of accepted strings is L. Justify your answer. [5 marks]

(*b*) Explain what is meant by a *regular expression* $\mathbf{r}$ over an alphabet Σ and by the language $L(\mathbf{r})$ determined by $\mathbf{r}$. [6 marks]

If a regular expression $\mathbf{r}$ does not contain any occurrence of the symbol $\emptyset$, is it possible for $L(\mathbf{r})$ to be empty? [2 marks]

Explain why it is always possible, given a regular expression $\mathbf{r}$ over Σ, to find a regular expression $\sim\mathbf{r}$ with the property that $L(\sim\mathbf{r})$ is the set of all strings over Σ that are not in $L(\mathbf{r})$. Any standard results you use should be carefully stated, but need not be proved. [7 marks]

10 Structured Hardware Design

In a specialist data communication network, digital data is sent between nodes using air pressure in pipes. Each node has a small number of pipes leading to each of its near neighbours. Each pipe may be either at atmospheric pressure or at some slightly higher pressure under control of, and detected by, transducers, fitted at each end of every pipe. The time for a pressure change to propagate down a given pipe depends on the length of the pipe and can vary from 1 to 10 milliseconds. Variations in the manufacturing of the transducers can sometimes add up to a further millisecond to the time taken to change the pressure or detect that it has changed.

For peer-to-peer data transfer, nine pipes run between each connected neighbour. They are used as eight data pipes and one clock pipe, with data on the eight data pipes being latched into a broadside register at the receiving end on each positive pressure change on the clock pipe.

(*a*) Design a timing diagram for reliable, continuous data transfer between a pair of nodes. [5 marks]

(*b*) What is the approximate, maximum, continuous data transfer rate achievable between adjacent nodes? (Assume all pipes between any pair of nodes are roughly the same length and allow a reasonable timing margin for safety.) [5 marks]

A data packet structure is now imposed on the system, whereby packets of 8 bytes of data may be sent by a node to a neighbour from time to time. Pipes are used bi-directionally, but only changing direction between packets. All pipes are to be at atmospheric pressure when there is no data to send.

(*c*) Suggest how the start and end of a packet can be determined. [5 marks]

(*d*) What will happen if both ends of a link decide to transmit to each other at the same time? Should an extra pipe be used to help solve this problem? [5 marks]

END OF PAPER