COMPUTER SCIENCE TRIPOS Part II (General)
DIPLOMA IN COMPUTER SCIENCE

Monday 2 June 2003 1.30 to 4.30

Paper 10 (Paper 1 of Diploma in Computer Science)

Answer **five** *questions.*

Submit the answers in five **separate** *bundles, each with its own cover sheet. On each cover sheet, write the numbers of* **all** *attempted questions, and circle the number of the question attached.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 Digital Electronics

(*a*) An n-bit decoder is a combinational circuit with n inputs and 2^n outputs. For each possible assignment of values to inputs there is a corresponding output which is set to "1" when and only when that assignment is made to the inputs. Give a design for a 3-bit decoder. [4 marks]

(*b*) Ignoring inverters, how many gates are required for your design? How many are required for an n-bit decoder? [2 marks]

(*c*) An n-bit priority encoder has $2^n - 1$ inputs and n outputs. The inputs $x_1, x_2 \ldots x_{2^n-1}$ are ordered in priority with x_j having higher priority than x_i if $j > i$. The outputs $a_{n-1}, a_{n-2} \ldots a_0$, interpreted as an unsigned integer, denote the highest priority input asserted high.
Give a design for a 3-bit priority encoder. [6 marks]

(*d*) Give *two* designs for a combinational circuit which has K ordered inputs and K corresponding outputs where the only output asserted (if any) is the one corresponding to the asserted input with the highest priority. [6 marks]

(*e*) Which design is better, and why? [2 marks]

2 Computer Design

The way in which an instruction's operands are specified is dependent on the type of internal storage in the processor. Processors which have no internal storage use memory.

(*a*) Describe the advantages of including some form of internal storage, such as an accumulator, in the processor. [5 marks]

(*b*) Describe *three* types of internal storage. In each case describe the format of an addition instruction. Indicate the presence of any implicit operands where necessary. [7 marks]

(*c*) Describe how data-forwarding improves performance in a pipelined load/store architecture (RISC). What common characteristic of programs often causes this performance improvement to be very significant? [8 marks]

3 Foundations of Programming

The following Java program has been written by a novice who is attempting to implement a tree-sort algorithm. This test program is intended to set up three nodes. The value fields of these nodes are to be written out in ascending order.

```
public class TreeSort
 { public static void main(String[] args)
    { Node tree = null;
      tree.put(8);   tree.put(16);   tree.put(4);
      System.out.println("Sorted values:  " + tree);
    }
 }

class Node
 { private int val;
   private Node left, right;

   public Node(int n)
    { this.val = n;
      this.left = null;
      this.right = null;
    }

   public void put(int k)
    { if (this == null)
         this = new Node(k);          // Error noted here
      if (k < this.val)
         this.left = new Node(k);
      else
         this.right = new Node(k);
    }
 }
```

(*a*) The compiler reports a single error, complaining about the statement indicated. What is the problem? Explain why there is more to fixing the program than merely changing this statement. [5 marks]

(*b*) Making the minimum number of changes (which will include adding a `toString()` method to class `Node`), modify the program so that it works in the way you think the author intended. [10 marks]

(*c*) Provide for class `Node` a method `sum()` which returns the sum of the elements in the tree. [5 marks]

4 Data Structures and Algorithms

(*a*) In the first phase of heapsort, an initially random vector is rearranged to satisfy the heap structure constraints. Describe what these are, how the rearrangement is done, and prove that it can be done in $O(n)$ time, where n is the number of elements in the vector. [7 marks]

(*b*) Complete the description of heapsort and show that its worst case performance is $O(n \log n)$. [7 marks]

(*c*) How many element comparisons would your implementation use to sort the integers 1 to 8 if they were (*i*) initially in sorted order, and (*ii*) initially in reverse sorted order? Explain how you obtained your answers. [6 marks]

5 Comparative Programming Languages

It has been said that "you can only become a really effective user of a programming language if you have a good understanding of how all its features are implemented". Discuss to what extent this is true. Your answer should include consideration of the representation of data, function calling mechanisms, space allocation and the implementation of object oriented features for a variety of different languages. [20 marks]

6 Operating System Foundations

(*a*) (*i*) What is meant by the *address space* of a process?

(*ii*) Give an example of how a 32-bit address space might be allocated between the various components of user and operating system (OS) code and data. You may assume that the OS occupies half the address space of every process.

(*iii*) Why does the OS region containing memory-mapped I/O interfaces need to be distinguished? What is the alternative to memory-mapped I/O?

(*iv*) How might the fact that much OS code must be permanently resident in memory be used to advantage?

(*v*) How is the OS protected from user level code at runtime?

(*vi*) How is the OS executed synchronously via calls from user level?

[13 marks]

(*b*) In what way did the provision of a protected address space per process affect the development of operating systems? Describe the hardware and software support for the development you mention in some detail. [7 marks]

7 Numerical Analysis I

(*a*) For IEEE Double Precision $\beta = 2$, $p = 53$, $e_{\text{min}} = -1022$, $e_{\text{max}} = 1023$. Explain the meaning of these parameters and deduce the number of bits required to store the *sign*, *exponent* and *significand*. How many bytes are required in total? [5 marks]

(*b*) What is the *hidden bit* and what is its value for *normalised numbers*, and for *denormal numbers*? [2 marks]

(*c*) Define *machine epsilon* ϵ_m. What is its value for IEEE Double Precision? [3 marks]

(*d*) Suppose $f(x) = O(1)$, $f'(x) = O(1)$ and

$$\frac{f(x+h) - f(x)}{h}$$

is to be used with IEEE Double Precision to estimate $f'(x)$ and $f''(x)$. State what value of h you would use in each case, and what absolute accuracy (as a power of 2) you would expect to achieve. [4 marks]

(*e*) Special purpose floating-point hardware is to be designed with the following specification. Each number is to occupy 6 bytes but otherwise obey the principles of IEEE arithmetic as far as possible. The arithmetic must be sufficiently accurate that second derivatives can be computed to an absolute accuracy of 10^{-3} if $f(x) = O(1)$, $f'(x) = O(1)$. Deduce the parameters of this arithmetic. [Hint: $10^{-3} \simeq 2^{-10}$ is sufficiently accurate.] [6 marks]

8 Mathematics for Computation Theory

Let A be a non-empty set. Define the *identity relation* Δ_A on A. [1 mark]

A *pre-order* on A is a relation R on A such that

(i) $\forall a \in A, (a, a) \in R$;

(ii) $(a, b) \in R, (b, c) \in R \Rightarrow (a, c) \in R$.

Using a similar notation, specify additional conditions:

(iii), that must be satisfied in order that R be a partial order on A;

(iv), that in addition to (iii) must be satisfied in order that R be a total order on A. [2 marks]

Express conditions (i)–(iv) in terms of relations only (i.e. without reference to elements of A). [3 marks]

Suppose R is a pre-order on A. Let

$$S = \{(a, b) \mid (a, b) \in R \quad \mathit{and} \quad (b, a) \in R\}.$$

Show that S is an equivalence relation on A. [4 marks]

Let $\frac{A}{S}$ be the set of S-equivalence classes. Write $[a]$ for $\{x \in A \mid (a, x) \in S\}$.

Define relation $\leqslant$ on $\frac{A}{S}$ as follows:

$$[a] \leqslant [b] \quad \text{iff} \quad (a, b) \in R.$$

Show that $\frac{A}{S}$ is partially ordered by $\leqslant$. [4 marks]

Let Z be the set of integers. Define the relation R on Z as follows:

$$\{(x, y) \in Z \times Z \mid \exists\, q \in Z \ s.t. \ y = xq\}.$$

Show that R is a pre-order on Z but *not* a partial order. Describe the derived partially ordered set $(\frac{Z}{S}, \leqslant)$. [4 marks]

What are the maximal and minimal elements in $(\frac{Z}{S}, \leqslant)$? [2 marks]

9 Computation Theory

What is meant by a *register machine*? Explain the action of a register machine program. [6 marks]

What does it mean for a partial function $f(x_1, \ldots, x_n)$ of n arguments to be *register machine computable*? [3 marks]

Design register machines to compute the following functions.

$$f(x_1, x_2) = x_1 + x_2$$ [2 marks]

$$g(x_1) = \begin{cases} 42 & \text{if } x_1 > 0 \\ \text{undefined} & \text{otherwise} \end{cases}$$ [2 marks]

$$h(x_1) = 2^{x_1}$$ [4 marks]

Give an example of a function that is not register machine computable, stating clearly any well-known results you use. [3 marks]

10 Computer Graphics and Image Processing

(*a*) Pick *two* of these three colour spaces: Lab, CMYK, HLS. For each of your chosen two colour spaces, explain what each of the dimensions represents and for what uses the colour space is best suited. [6 marks]

(*b*) Describe a run-length encoding method for greyscale images. [6 marks]

(*c*) The following is a Bezier curve drawing algorithm which includes bounding box clipping. Provide pseudocode for the functions `InBoundingRect` and `DrawUnclippedBezier`.

```
function DrawClippedBezier(float x1, y1, x2, y2, x3, y3, x4, y4)
begin
  if NearlyStraight(x1, y1, x2, y2, x3, y3, x4, y4)
  then DrawClippedLine(x1, y1, x4, y4)
  else begin
      r = InBoundingRect(x1, y1, x2, y2, x3, y3, x4, y4) ;
      if(r==0) then DrawUnclippedBezier(x1, y1, x2, y2, x3, y3, x4, y4);
      if(r==1) then begin
          DrawClippedBezier(x1,y1, (x1+x2)/2,(y1+y2)/2,
            (x1+2*x2+x3)/4,(y1+2*y2+y3)/4,
            (x1+3*x2+3*x3+x4)/8,(y1+3*y2+3*y3+y4)/8);
          DrawClippedBezier((x1+3*x2+3*x3+x4)/8,(y1+3*y2+3*y3+y4)/8,
            (x2+2*x3+x4)/4,(y2+2*y3+y4)/4,(x3+x4)/2,(y3+y4)/2,x4,y4);
      end ;
      if(r==2) then return ;
  end ;
end;
```

Notes: The bounding rectangle is defined by the four (global) floating point variables `left`, `right`, `top`, and `bottom`. You may assume that we have two line drawing functions available `DrawClippedLine` and `DrawUnclippedLine`. The former draws a line having first clipped it to the bounding rectangle, the latter just draws a line without regard for the bounding rectangle (which should therefore only be used if the programmer has assured him- or herself that the line will not extend beyond the bounding rectangle). The two functions `DrawClippedBezier` and `DrawUnclippedBezier` do the same for Bezier curves. The function `NearlyStraight` returns `true` if the Bezier curve lies within half a pixel of a straight line from its first to its last point along its entire length, otherwise it returns `false`.

[8 marks]

11 Introduction to Security

(*a*) Explain the difference between mandatory and discretionary access control. [4 marks]

(*b*) (*i*) Explain the purpose and operation of cipher-block chaining (CBC). [4 marks]

(*ii*) Explain how to decrypt a message in CBC. [4 marks]

(*c*) To protect her interview partners, a journalist needs to ensure that what she records with her digital camera cannot be viewed by anyone before she returns to her home country. You were asked to design for her a camera that encrypts recordings immediately before they are stored on tape. The question arises, how to handle the encryption key. If it is stored in the camera, it could be extracted if the hardware were confiscated and analysed. A key memorised by the user might be obtained using coercion, so this is not a suitable solution either.

Suggest *two* alternative convenient ways of arranging the encryption inside the camera such that decryption of the tape is possible only on the journalist's home computer. [8 marks]

12 Software Engineering and Design

(*a*) Describe the roles of (*i*) use case diagrams and (*ii*) collaboration diagrams in a UML design. [4 marks]

(*b*) How would the role of *each* vary in a project managed according to (*i*) the waterfall model and (*ii*) user-centred design? [4 marks]

(*c*) Which of the management models in part (*b*) best describes the Cambridge University CAPSA project? Why? [2 marks]

(*d*) What information is carried between these diagrams in the design process? Illustrate with reference to a researcher requesting University approval to purchase a disk drive from an online catalogue. Include suitable diagrams in your answer. [10 marks]

END OF PAPER