2002 Paper 9 Question 6

Types

(*a*) Give the typing rules for the polymorphic lambda calculus (PLC). [5 marks]

(*b*) Let $\mathit{prod}(\alpha_1, \alpha_2)$ denote the PLC type $\forall\alpha((\alpha_1 \to (\alpha_2 \to \alpha)) \to \alpha)$. Explain how it behaves like the ML product type $\alpha_1 * \alpha_2$. To do so, you should give PLC expressions *pair*, *fst* and *snd* of types

$$\begin{aligned} & \forall\alpha_1(\forall\alpha_2(\alpha_1 \to (\alpha_2 \to \mathit{prod}(\alpha_1, \alpha_2)))), \\ & \forall\alpha_1(\forall\alpha_2(\mathit{prod}(\alpha_1, \alpha_2) \to \alpha_1)), \\ \text{and} \quad & \forall\alpha_1(\forall\alpha_2(\mathit{prod}(\alpha_1, \alpha_2) \to \alpha_2)) \end{aligned}$$

respectively, corresponding to the ML polymorphic pairing and projection functions

```
fn x1 => fn x2 => (x1, x2) ,
fn (x1, x2) => x1 ,
and fn (x1, x2) => x2 .
```

Give proofs for the typing of *pair* and *fst*, and explain the beta-conversion properties of $\mathit{fst}\,\tau_1\,\tau_2(\mathit{pair}\,\tau_1\,\tau_2\,M_1\,M_2)$, for any PLC types τ_1, τ_2 and terms M_1, M_2. [10 marks]

(*c*) Is it always the case that a PLC term M of type $\mathit{prod}(\tau_1, \tau_2)$ is beta-convertible to $\mathit{pair}\,\tau_1\,\tau_2(\mathit{fst}\,\tau_1\,\tau_2\,M)(\mathit{snd}\,\tau_1\,\tau_2\,M)$? Justify your answer. [Hint: consider terms with free variables.] [5 marks]