2002 Paper 9 Question 11

Numerical Analysis II

Consider the alternative formulae

$$y_{n+1} = y_n + hf(x_n, y_n) + O(h^2) \qquad (1)$$

$$y_{n+1} = y_{n-1} + 2hf(x_n, y_n) + O(h^3) \qquad (2)$$

applied to the ODE

$$y' = -5y, \quad y(0) = 1$$

using $h = 0.1$ in each case.

(a) Define the terms *local error* and *order* for an ODE formula. What is the *order* of each of the methods (1) and (2)? [2 marks]

(b) Giving answers to 2 significant decimal digits of accuracy, compute the solution of the ODE for $x_n = 0, 0.1, 0.2, \ldots$ 1.0 for each method. Tabulate your answers. The exact solutions to 2 significant digits are:

1.0, 0.61, 0.37, 0.22, 0.14, 0.082, 0.050, 0.030, 0.018, 0.011, 0.0067

Assume the exact value of $y(0.1)$ for method (2). [7 marks]

(c) Which method is more accurate initially and why? Explain the behaviour of each method as x increases. [3 marks]

(d) Solve the ODE. Find a general term for y_n in method (1) and show that the absolute error in (1) will be small when n is large. Without performing any further calculations, how do you expect the absolute error in method (2) to behave when n is large? [5 marks]

(e) Discuss briefly the suitability of formulae (1) and (2) as predictors for predictor–corrector methods in respect of *order* and *stability.* [3 marks]