Numerical Analysis II

Consider the alternative formulae

\[ y_{n+1} = y_n + hf(x_n, y_n) + O(h^2) \]  \hspace{1cm} (1)
\[ y_{n+1} = y_{n-1} + 2hf(x_n, y_n) + O(h^3) \]  \hspace{1cm} (2)

applied to the ODE

\[ y' = -5y, \quad y(0) = 1 \]

using \( h = 0.1 \) in each case.

(a) Define the terms local error and order for an ODE formula. What is the order of each of the methods (1) and (2)? \hspace{1cm} [2 marks]

(b) Giving answers to 2 significant decimal digits of accuracy, compute the solution of the ODE for \( x_n = 0, 0.1, 0.2, \ldots 1.0 \) for each method. Tabulate your answers. The exact solutions to 2 significant digits are:

1.0, 0.61, 0.37, 0.22, 0.14, 0.082, 0.050, 0.030, 0.018, 0.011, 0.0067

Assume the exact value of \( y(0.1) \) for method (2). \hspace{1cm} [7 marks]

(c) Which method is more accurate initially and why? Explain the behaviour of each method as \( x \) increases. \hspace{1cm} [3 marks]

(d) Solve the ODE. Find a general term for \( y_n \) in method (1) and show that the absolute error in (1) will be small when \( n \) is large. Without performing any further calculations, how do you expect the absolute error in method (2) to behave when \( n \) is large? \hspace{1cm} [5 marks]

(e) Discuss briefly the suitability of formulae (1) and (2) as predictors for predictor–corrector methods in respect of order and stability. \hspace{1cm} [3 marks]