

## 2002 Paper 9 Question 1

### Denotational Semantics

- (a) Let  $A$  be a set. Let  $R$  consist of a set of pairs  $(X_0, a)$  where  $X_0$  is a finite subset of  $A$  and  $a \in A$ . Define a function on the powerset of  $A$ ,

$$\widehat{R} : \mathcal{P}(A) \rightarrow \mathcal{P}(A)$$

by

$$\widehat{R}(X) = \{ a \mid \exists X_0 \subseteq X. (X_0, a) \in R \}$$

for  $X \subseteq A$ . Prove that the function  $\widehat{R}$  is a continuous function on the domain  $(\mathcal{P}(A), \subseteq)$ . [6 marks]

- (b) Describe how to construct the function  $\text{cpo}((D \rightarrow E), \sqsubseteq)$  of two cpos  $(D, \sqsubseteq_D)$  and  $(E, \sqsubseteq_E)$ . Prove that  $((D \rightarrow E), \sqsubseteq)$  is a cpo. (You may use facts about least upper bounds provided you state them clearly.) [7 marks]

- (c) Exhibit two terms of PCF which are contextually equivalent and yet have distinct denotations in the domain  $(\mathbb{B}_\perp \rightarrow (\mathbb{B}_\perp \rightarrow \mathbb{B}_\perp)) \rightarrow \mathbb{B}_\perp$  where  $\mathbb{B} = \{true, false\}$  is the set of truth values. Exhibit the domain element on which the two denotations differ. [7 marks]