

## 2002 Paper 1 Question 9

### Programming in Java

Observe that the matrix equation

$$\begin{pmatrix} f_{n+2} \\ f_{n+1} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} f_{n+1} \\ f_n \end{pmatrix}$$

together with the initial conditions  $f_0 = f_1 = 1$  defines the sequence of Fibonacci numbers.

Now observe that to compute  $f_n$  you need to multiply the vector  $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$  by the matrix shown above raised to the power  $n - 1$ . In the special case where  $n$  is one bigger than a power of 2 this can be done by repeatedly squaring the matrix.

Write a Java program that will use this idea to compute  $f_{1025}$  given that  $1024 = 2^{10}$ . Your matrix multiplying or squaring code should be arranged to be potentially re-usable for matrices of arbitrary size: code that works only for 2 by 2 matrices is not acceptable.

Credit will be given for the clarity with which you present your design, the coherence of the explanation of how and why it works and for your comments about the cost or efficiency of your program.

[20 marks]