Foundations of Computer Science

Consider the following ML declarations, involving binary trees:

\[
\text{datatype 'a tree} = \text{Lf} \\
\quad | \text{Br of 'a * 'a tree * 'a tree};
\]

\[
\text{exception E;}
\]

\[
\text{fun path Lf = raise E} \\
| \text{path (Br(v,t1,t2))} = \begin{cases} 
\text{if v=7 then []} \\
\text{else 1 :: path t1} \\
\text{handle E => 2 :: path t2;}
\end{cases}
\]

(a) The function \text{path} returns a path (a list of 1s and 2s) to an occurrence of the number 7 in the tree. Carefully explain how \text{path} works, taking the tree shown below as an example and indicating which occurrence of 7 will be found. \[5\text{ marks}\]

(\text{b}) Code the function \text{paths}, which returns the list of all paths to 7s in a binary tree. \[5\text{ marks}\]