

2002 Paper 12 Question 12

Complexity Theory

- (a) Give a precise definition of what is meant by the statement that a decision problem A is *polynomial-time reducible* to a decision problem B . [2 marks]
- (b) Consider the following three decision problems on graphs.

Connect—the collection of graphs G such that there is a path between any two vertices of G .

Hamilton—the collection of graphs that contain a Hamiltonian cycle.

non-3-colour—the collection of graphs that are not 3-colourable.

For each of the following statements, state whether it is true, false or an unresolved open question. Give a brief justification for your answer.

- (i) **Connect** is decidable by a polynomial time algorithm.
- (ii) **Hamilton** is decidable by a polynomial time algorithm.
- (iii) **non-3-colour** is decidable by a polynomial time algorithm.
- (iv) **Connect** is polynomial-time reducible to **Hamilton**.
- (v) **Hamilton** is polynomial-time reducible to **non-3-colour**.
- (vi) **non-3-colour** is polynomial-time reducible to **Connect**.

[3 marks each]