Numerical Analysis II

(a) A cubic spline over knots \( x_1, x_2, \ldots x_n \) is defined by

\[
\phi(x) = \frac{(x - x_j)y_{j+1} + (x_{j+1} - x)y_j}{d_j} - \frac{(x - x_j)(x_{j+1} - x)(d_j + x_{j+1} - x)\mu_j + (d_j + x - x_j)\mu_{j+1}}{6d_j}
\]

for \( x \in [x_j, x_{j+1}] \) where \( d_j = x_{j+1} - x_j \). The spline is continuous in its first and second derivatives.

(i) Find \( \phi(x_j) \). [2 marks]

(ii) Find formulae for \( \phi'(x_j) \) and \( \phi'(x_{j+1}) \) for \( x \in [x_j, x_{j+1}] \). [4 marks]

(iii) What is \( \phi''(x_j) \)? [2 marks]

(b) Form a set of equations for computing the unknowns \( \{\mu_j\} \), specifying suitable end conditions to simplify these equations. [10 marks]

(c) What are the important properties of these equations with respect to their numerical solution? [2 marks]