

2001 Paper 6 Question 1

Data Structures and Algorithms

- (a) State what is meant by a *directed graph* and a *strongly connected component*. Illustrate your description by giving an example of such a graph with 8 vertices and 12 edges that has three strongly connected components. [5 marks]
- (b) Describe, in detail, an algorithm to perform a depth-first search over such a graph. Your algorithm should attach the discovery and finishing times to each vertex and leave a representation of the depth-first spanning tree embedded within the graph. [5 marks]
- (c) Describe an $O(n)$ algorithm to discover all the strongly connected components of a given directed graph and explain why it is correct. You may find it useful to use the concept of the forefather $\phi(v)$ of a vertex v which is the vertex, u , with highest finishing time for which there exists a (possibly zero length) path from v to u . [10 marks]