

## 2001 Paper 5 Question 9

### Semantics of Programming Languages

- (a) The integer expressions  $E$  of a programming language are given by

$$E ::= n \mid X \mid -E \mid E + E$$

where  $n$  ranges over integer constants and  $X$  ranges over identifiers. Explain the principle of *structural induction* for proving that some property  $\Phi(E)$  holds for all integer expressions  $E$ . [5 marks]

- (b) Taking *states* to be finite partial functions mapping identifiers to integer constants, define a relation

$$E, s \Downarrow n$$

giving the result  $n$  (if any) of evaluating integer expression  $E$  in state  $s$ .

[7 marks]

- (c) Use structural induction to prove that if  $E, s \Downarrow n_1$  and  $E, s \Downarrow n_2$  both hold, then  $n_1 = n_2$ . [Hint: Consider the property  $\Phi(E)$  given by  $\forall s, n_1, n_2 ((E, s \Downarrow n_1) \ \& \ (E, s \Downarrow n_2) \Rightarrow n_1 = n_2)$ .] [7 marks]

- (d) What property of the pair  $E, s$  ensures that there is some  $n$  for which  $E, s \Downarrow n$  holds? [1 mark]