

2001 Paper 5 Question 10

Foundations of Functional Programming

- (a) Write a pure lambda-expression that will act as a fixed-point operator Y such that the identity $Y f = f(Y f)$ will hold. [6 marks]
- (b) Write pure lambda-expressions that define functions P , A and D such that $A (P x y) = x$ and $D (P x y) = y$. Observe that P can be thought of as creating a 2-tuple and A and D then act as selectors that can retrieve the two components. [7 marks]
- (c) Using the two above lambda-expressions it is possible to express mutual recursion between two functions, say f and g . This can be done by using Y to help find the value of $(P f g)$ the tuple whose elements are f and g . Using the artificial and rather silly example [the example will never terminate since it has no stopping condition!]

$$\begin{aligned} f x &= g (f (g x)) \\ \text{AND } g x &= g (f x) \end{aligned}$$

show how to construct a pure lambda expression that would evaluate

$$(f g)$$

[7 marks]