

2001 Paper 2 Question 7

Regular Languages and Finite Automata

- (a) Suppose that L is a language over a finite alphabet Σ with the property that for each number $\ell \geq 1$ there is some string w in L with $\text{length}(w) \geq \ell$ such that no matter how w is split up into three pieces $w = u_1vu_2$ with $\text{length}(u_1v) \leq \ell$ and $\text{length}(v) \geq 1$, there is some $n \geq 0$ for which $u_1v^n u_2$ is not in L . Prove that L cannot be a regular language. [12 marks]
- (b) State, with justification, whether each of the following languages over $\Sigma = \{a, b\}$ is regular.
- (i) $L_1 = \{ww \mid w \in \Sigma^*\}$. [5 marks]
- (ii) $L_2 = \{wvw \mid v, w \in \Sigma^*\}$. [3 marks]