2001 Paper 12 Question 10

Numerical Analysis II

(*a*) A cubic spline over knots $x_1, x_2, \ldots x_n$ is defined by

$$\phi(x) = \frac{(x - x_j)y_{j+1} + (x_{j+1} - x)y_j}{d_j} - \frac{(x - x_j)(x_{j+1} - x)\{(d_j + x_{j+1} - x)\mu_j + (d_j + x - x_j)\mu_{j+1}\}}{6d_j}$$

for $x \in [x_j, x_{j+1}]$ where $d_j = x_{j+1} - x_j$. The spline is continuous in its first and second derivatives.

(*i*) Find $\phi(x_j)$. [2 marks]

(*ii*) Find formulae for $\phi'(x_j)$ and $\phi'(x_{j+1})$ for $x \in [x_j, x_{j+1}]$. [4 marks]

(*iii*) What is $\phi''(x_j)$? [2 marks]

(*b*) Form a set of equations for computing the unknowns $\{\mu_j\}$, specifying suitable end conditions to simplify these equations. [10 marks]

(*c*) What are the important properties of these equations with respect to their numerical solution? [2 marks]