COMPUTER SCIENCE TRIPOS Part IA

Tuesday 5 June 2001 1.30 to 4.30

Paper 2

Answer the question in Section A, **one** *question from each of Sections B and C, and* **two** *questions from Section D.*

Submit the answers in five **separate** *bundles, each with its own cover sheet. On each cover sheet, write the numbers of* **all** *attempted questions, and circle the number of the question attached.*

Write on **one** *side of the paper only.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 Multi-part question

Answer all five parts.

(a) "The ML type system ensures that every program does what it is supposed to do." Write *three* sentences, each either supporting or refuting this statement. [4 marks]

(b) A lot of large software projects fail. Many companies described Y2K bug fixing as a substantial project. Why then did the expected chaos not materialise in January 2000? [4 marks]

(c) Describe the software engineering principles of loop design. Illustrate your answer using the example of a loop that reverses the order of the first N elements of the array A. [4 marks]

(d) Draw a picture of a deterministic finite automaton with set of input symbols $\{a, b\}$ whose language of accepted strings consists of all strings containing an odd number of occurrences of the symbol a. [4 marks]

(e) Bluetooth is a radio technology where tiny nodes replace lengths of wire or cable between items of equipment with little or no modification to the design of the equipment. Sketch a block diagram of a typical node and write *three* sentences describing three of the main blocks. [4 marks]

SECTION B

2 Digital Electronics

A resource is managed by a system that allocates it to at most one of four requesters for one cycle at a time. The resource is allocated in a "round robin" fashion, so that for example if requester 2 last had the resource, requester 3 has highest priority to acquire it, followed by, in order, requesters 0, 1 and 2. If no one requests the resource it is not allocated, but the system still remembers who last had the resource.

(*a*) Define input and output variables for the system. [3 marks]

(*b*) Describe state variables for the system. [3 marks]

(*c*) Provide equations for the state control and outputs. [10 marks]

(*d*) Find a minimal sum of products expression for *one* of the outputs. [4 marks]

3 Digital Electronics

Consider two numbers X and Y, each represented by n boolean variables $x_{n-1}x_{n-2}\ldots x_0$ and $y_{n-1}y_{n-2}\ldots y_0$ in the usual way so that for example $X = \sum_{i=0}^{n-1} 2^i x_i$.

(*a*) Design a full adder to find $Z = X + Y$ in the case where $n = 2$. If each gate has a delay τ, how quickly is the result of the addition available after the inputs are presented? [5 marks]

(*b*) Estimate a rough upper bound on the number of gates required to build a full adder in combinational logic when $n = 4$. [3 marks]

(*c*) Describe *two* techniques for building adders which reduce gate count. [7 marks]

(*d*) Design a full multiplier to find $W = X \times Y$ for the case where $n = 2$. [5 marks]

SECTION C

4 Probability

(a) Solve the following inhomogeneous difference equation:

$$u_n = 2(u_{n-1} + 3) \qquad \text{given that} \quad u_1 = 0$$

It may be assumed that $n \geqslant 1$. [5 marks]

A hardware device generates streams of ternary digits. Within a stream, each digit is equiprobably 0, 1 or 2. A stream ends as soon as each digit has been seen at least once. A stream may be as short as three digits (for example 201) but is usually rather longer (for example 1110102).

(b) Clearly there are three ways in which the first k digits of a stream may all be the same. What is the probability that the first k digits are all the same? [1 mark]

(c) By using the difference equation above, or otherwise, determine the number of ways in which the first k digits of a stream could comprise exactly two of the three available digits. [5 marks]

(d) What is the probability that the first k digits comprise exactly two of the three available digits? [1 mark]

(e) For $r \geqslant 2$, what is the probability that a stream is r digits long? [3 marks]

(f) What is the expected length of a stream? [5 marks]

Hint: It may be useful to note that

$$\sum_{r=1}^{\infty} r x^{r-1} = \frac{1}{(1-x)^2} \qquad \text{if} \quad 0 \leqslant x < 1$$

5 Probability

Candidates for a certain first-year University Examination offer four subjects. At their first meeting the Examiners agree that, within each subject, they will sort the candidates into descending order of marks and those ranked in the top third will be awarded informal Firsts for that subject.

To save effort, the Examiners agree to mark each subject simply by throwing a fair die once for each candidate and awarding the outcome, a mark in the range 1 to 6. They reason, correctly, that 5 should be the threshold mark for a First since approximately one-third of the candidates are likely to score 5 or 6.

Noting that 5 is the threshold mark for a First in an individual subject and that each candidate offers four subjects, the Examiners agree further that 20 marks (4×5) should be the threshold mark for a First overall in the Examination. They reason, incorrectly, that using 20 as the threshold about one-third of the candidates will gain Firsts overall.

When they meet after the Examination, the Examiners are astonished to discover that, using a threshold of 20 marks, considerably fewer than one-third of the candidates gain Firsts.

(a) Assuming that dice throws are independent, what is the probability that a candidate obtains 20 or more marks overall? [12 marks]

(b) To what value should the Examiners lower the threshold mark for a First overall to ensure that approximately one-third of the candidates gain Firsts? [5 marks]

(c) What is the probability that a candidate obtains or exceeds this lower threshold? [1 mark]

(d) Suppose the candidates were assessed more conventionally (on perceived ability) but still awarded integer marks in the range 1 to 6 for each subject. Suppose further that the threshold mark of 5 for a First again results in about one-third of the candidates in any individual subject being awarded a First. Discuss whether it would be reasonable now for the Examiners to assume that an overall threshold of 20 would lead to approximately one-third of the candidates gaining Firsts overall. [2 marks]

SECTION D

6 Professional Practice and Ethics

(*a*) What kinds of law can be used in protecting intellectual property? [6 marks]

(*b*) What problems are faced with each of these kinds of law in protecting software as a kind of intellectual property (i.e. creating proprietary software)? [4 marks]

(*c*) What is the ethical justification for protection of software as a kind of intellectual property? [3 marks]

(*d*) What arguments are used to oppose this justification? [3 marks]

(*e*) What alternative is there to proprietary software? [2 marks]

(*f*) How could computer professionals support themselves without proprietary software? [2 marks]

7 Regular Languages and Finite Automata

(*a*) Suppose that L is a language over a finite alphabet Σ with the property that for each number $\ell \geqslant 1$ there is some string w in L with $length(w) \geqslant \ell$ such that no matter how w is split up into three pieces $w = u_1 v u_2$ with $length(u_1 v) \leqslant \ell$ and $length(v) \geqslant 1$, there is some $n \geqslant 0$ for which $u_1 v^n u_2$ is not in L. Prove that L cannot be a regular language. [12 marks]

(*b*) State, with justification, whether each of the following languages over $\Sigma = \{a, b\}$ is regular.

(*i*) $L_1 = \{ww \mid w \in \Sigma^*\}$. [5 marks]

(*ii*) $L_2 = \{wvw \mid v, w \in \Sigma^*\}$. [3 marks]

8 Software Engineering I

(*a*) State the advantages and disadvantages of the evolutionary model of software development. [8 marks]

(*b*) Is it more, or less, suitable than the waterfall model for safety-critical projects? Justify your answer. [8 marks]

(*c*) If you were managing the evolution of a safety-critical product, what special steps would you take for risk reduction or due diligence reasons? [4 marks]

9 Software Engineering II

(*a*) Describe, with examples, how the choice of programming language, programming tools and libraries can affect the reliability of the software developed using them. [5 marks]

(*b*) Consider the following pair of ML function declarations:

```
fun takew p [] = []
  | takew p (x::xs) = if p x then x :: takew p xs else [];

fun dropw p [] = []
  | dropw p (x::xs) = if p x then dropw p xs else x::xs;
```

Prove `(takew p xs) @ (dropw p xs) = xs` using induction. (Assume that function `p` always terminates.) [8 marks]

(*c*) You have been asked to specify some banking software. A bank account has a *balance* and an overdraft *limit*, subject to the constraints $limit \geqslant 0$ and $balance + limit \geqslant 0$.

(*i*) Write a Z schema to specify the state of a bank account. [2 marks]

(*ii*) Write a Z schema for the operation to withdraw a given positive *amount* from the account. [5 marks]

10 Structured Hardware Design

(*a*) Explain the basic operation of the *parallel port* found on most computers. Include a description of the main signals and give their purpose. [4 marks]

(*b*) Similarly, explain the basic operation of the RS232 *serial port* found on most computers. [4 marks]

(*c*) What sets the maximum rate of transfer of data for each type of port? What happens if the receiver cannot consume data as quickly as the source would like to send it? [4 marks]

(*d*) Both types of port allow data to flow from one piece of equipment to another and these items will normally have independent clock domains. How can synchronisation be achieved? [4 marks]

(*e*) When might it be sensible to communicate data using a large number of wires in parallel but with each one behaving more like an individual serial port? [4 marks]

END OF PAPER