2000 Paper 2 Question 4

Probability

An ordinary fluorescent light tube exhibits a lack of memory property in that its life expectancy does not depend on how long it has been working. A typical tube may be expected to work for another 5040 hours no matter how long it has been working so far.

If a room contains eight such tubes and all are working, one may expect the first failure after 630 hours (5040/8). If the dud tube is *not* replaced, after how many more hours may one expect the second failure and (again assuming no replacement) after how many more may one expect the third failure? [2 marks]

Consider a meeting room which is equipped with four light fittings, each equipped with two such tubes. The management has decided that the room lighting is acceptable provided at least one tube in each pair is working. As soon as a second tube fails in any one fitting, a maintenance crew replaces *all* dud tubes in the room. Starting with eight working tubes, the crew may be called out as early as the second failure or as late as the fifth failure.

Let X be a random variable whose value r is the number of dud tubes at the moment the maintenance crew is called out. Clearly $\mathrm{P}(X = 0) = \mathrm{P}(X = 1) = 0$. Determine the values of $\mathrm{P}(X = 2)$, $\mathrm{P}(X = 3)$, $\mathrm{P}(X = 4)$ and $\mathrm{P}(X = 5)$. Express all four results as fractions. It may be assumed that all fittings are permanently switched on and that tubes fail independently. [12 marks]

What is the expectation, $\mathrm{E}(X)$? [4 marks]

The management rounds the value of $\mathrm{E}(X)$ down to the nearest integer and uses the derived value for estimating the number of tubes that have to fail between successive call-outs of the maintenance crew and the time interval between such call-outs. What is this time interval in hours? [2 marks]