State the time complexity of the lookup and update operations for each of the following:

(a) association lists
(b) binary search trees
(c) functional arrays (implemented as binary trees)

Use $O$-notation and include both the average-case and worst-case complexity. [6 marks]

You are provided with the ML code for binary search trees, including the lookup and update operations. Use these operations to code a sorting function that works by repeatedly inserting elements of a list into a binary search tree, then converting the final binary search tree back into a list. [4 marks]

Consider the following methods of sorting a list:

(a) Locate the smallest element of the input. The output is this element followed by the result of recursively sorting the remaining elements.

(b) Take the first 16 elements of the input and sort them using special hardware. Sort the remaining elements recursively. The output is the result of merging the two sorted lists.

(c) Take the first 20% of the input elements. Sort them and the remaining 80% recursively. The output is the result of merging the two sorted lists.

For each of these methods, state with justification the worst-case complexity in terms of the number of comparisons. [10 marks]