2000 Paper 11 Question 8

Mathematics for Computation Theory

Let E, F be events over a finite alphabet S. Define the events $E + F$, EF and E^*. [3 marks]

Show that:

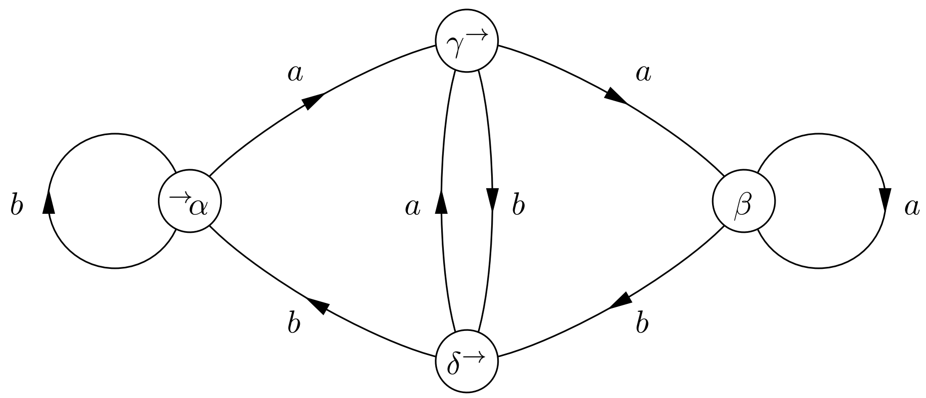
(a) $E^* = 1 + EE^*$

(b) $E(FE)^* = (EF)^*E$

[4 marks]

State Kleene's Theorem on the structure of events accepted by some Deterministic Finite Automaton (DFA). [1 mark]

Consider the following DFA:



Here α is the initial state, γ and δ the two accepting states. Show that the event accepted is

$$b^*a(a^*b\,b^*a)^*\{1 + a^*b\}.$$

[12 marks]

[Hint. If $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ is a partitioning of the transition matrix of a DFA so that A and D are square, then

$$M^* = \begin{pmatrix} (A + BD^*C)^* & A^*B(D + CA^*B)^* \\ D^*C(A + BD^*C)^* & (D + CA^*B)^* \end{pmatrix}$$

with the same partitioning. Partition the states in the order $\{\alpha, \beta\}, \{\gamma, \delta\}$. You need calculate only the upper right component of M^*.]