1999 Paper 8 Question 7

Optimising Compilers

Consider a first-order call-by-need language with identifiers x_i and expressions e_i (which can be of type `int` only) and function names A_i (built-in) and G (a single, possibly recursive, user-defined function of the form $G(x_1, \ldots, x_k) = e$) whose arguments and results are of type `int`.

Describe the basic concepts of strictness analysis. You should explain what space of abstract values you would use to model strictness properties of a function of k arguments, and give the abstract strictness values for *cond* and *plus* (respectively the ternary conditional function and the binary addition function). State how one can determine that "f is strict in its i^{th} argument" in terms of your abstract value for a function f and how the abstract strictness value for G is obtained. [10 marks]

As an alternative method of deriving strictness properties, it is proposed to use an effect-like system instead. Suppose f is a function of k arguments and S is a subset of $\{1, \ldots, k\}$. In such a system judgements *on functions* f are of the form

$$\Gamma \vdash f : \texttt{int}^k \stackrel{S}{\to} \texttt{int}$$

where Γ is a set of type assumptions on variables x. The above judgement is defined to be valid if, whenever f is applied and all argument expressions in argument positions S fail to terminate, then the call to f fails to terminate (such f are often called S-jointly strict). In the following t ranges over type and effect forms $\texttt{int}^k \stackrel{S}{\to} \texttt{int}$.

Give an inference rule (here called (SUB)) for judgements of the form $\Gamma \vdash f : t$ which captures the idea that "if f is S-jointly strict in argument positions S, then a call to it fails to terminate when applied to argument expressions which fail to terminate for a larger set of argument positions". [3 marks]

Give a suitable set of assumptions of the form

$$\Gamma_0 = \{plus : t_1, plus : t_2, cond : t_3, cond : t_4\}$$

which together with the (SUB) rule above enable one to deduce exactly the valid strictness judgements $\Gamma_0 \vdash f : t$ when f is *plus* or *cond*. [Hint: there are two t_i for both *plus* and *cond*.] [4 marks]

Give conditions on t in the judgement $\Gamma \vdash f : t$ which enables the claim "f is strict in its i^{th} argument" to be made. [3 marks]