Denotational Semantics

Suppose that $f : D \to D$ is a continuous function on a domain. What is meant by the least pre-fixed point, $\text{fix}(f)$, of $f$? [2 marks]

Show that $\text{fix}(f)$ exists and is in fact the least fixed point of $f$. [12 marks]

Suppose now that $E$ is another domain and $g : D \times E \to E$ a continuous function. Let $(d, e)$ be the least element of $D \times E$ satisfying

$$\begin{cases} 
  d = f(d) \\
  e = g(d, e)
\end{cases}$$

Prove that $d = \text{fix}(f)$. [6 marks]