1999 Paper 7 Question 2

Specification and Verification I

For $n \geqslant 0$, define $\Sigma(A, n)$ recursively by:

$$\Sigma(A,n) \;=\; \mathit{if}\ n=0\ \mathit{then}\ A(0)\ \mathit{else}\ \Sigma(A,n{-}1)+A(n)$$

For $m \geqslant 0$ and $n \geqslant 0$ define $\Sigma2(A, m, n)$ by:

$$\begin{aligned}\Sigma2(A,m,n) = \quad & \mathit{if}\ m=0 \\ & \mathit{then}\ \Sigma(A,n) \\ & \mathit{else} \\ & \ \mathit{if}\ m \leqslant n\ \mathit{then}\ \Sigma(A,n)-\Sigma(A,m{-}1)\ \mathit{else}\ 0\end{aligned}$$

Show that:

(*a*) $\forall n.\ 0 \leqslant n \ \Rightarrow\ \Sigma2(A,n,n) = A(n)$ [2 marks]

(*b*) $\forall m\ n.\ 0 \leqslant m \wedge m < n \ \Rightarrow\ \Sigma2(A,m,n) = A(m)+A(n)+\Sigma2(A,m{+}1,n{-}1)$ [6 marks]

Give a detailed proof using Floyd–Hoare logic that:

$\vdash\ \{\mathtt{N} = n\ \wedge\ n \geqslant 0\}$

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     M := 0; SUM := 0;
     WHILE M ⩽ N DO
       IF M=N
         THEN BEGIN SUM := SUM + A(N); M := M+1 END
         ELSE BEGIN SUM := SUM+A(M)+A(N); M := M+1; N := N-1 END
```

$\{\mathtt{SUM} = \Sigma(\mathtt{A}, n)\}$

[12 marks]