(a) Give $\lambda$-terms $Y$, $K$, $T$ and $I$ satisfying the following equalities for all terms $M$ and $N$:

\[
YM = M(YM) \\
KMN = M \\
TMN = NM \\
IM = M
\]

[4 marks]

(b) A $\lambda$-term is \textit{defined} if it has a head normal form. For each of the following terms, state whether or not it is defined, giving justification for your answer.

\[
Y \quad YK \quad YT \quad YI
\]

[8 marks]

(c) A $\lambda$-term $M$ is \textit{solvable} if there exist variables $x_1, \ldots, x_m$ and terms $N_1, \ldots, N_n$ such that

\[
(\lambda x_1, \ldots, x_m.M)N_1 \ldots N_n = I
\]

For those terms in (b) that are solvable, exhibit the variables and terms that establish this. For those that are not, explain why they are not solvable.

[8 marks]