# 1999 Paper 4 Question 9

## Numerical Analysis I

The mid-point rule can be expressed in the form

$$I_n = \int_{n-\frac{1}{2}}^{n+\frac{1}{2}} f(x)dx = f(n) + e_n$$

where

$$e_n = f''(\theta_n)/24$$

for some $\theta_n$ in the interval $(n - \frac{1}{2}, n + \frac{1}{2})$. Assuming that a formula for $\int f(x)dx$ is known, and using the notation

$$S_{p,q} = \sum_{n=p}^{q} f(n),$$

describe a method for estimating the sum of a slowly convergent series $S_{1,\infty}$, by summing only the first $N$ terms and estimating the remainder by integration.

[6 marks]

Assuming that $f''(x)$ is a positive decreasing function, derive an estimate of the error $|E_N|$ in the method. [5 marks]

Given

$$\int \frac{dx}{x(x+2)} = -\frac{1}{2}\log_e(1 + \frac{2}{x})$$

illustrate the method by applying it to

$$\sum_{n=1}^{\infty} \frac{1}{n(n+2)}.$$

Verify that $f''(x)$ is positive decreasing for large $x$, and estimate the integral remainder to be added to $S_{1,N}$. [You may assume $\log_e(1+\lambda) \simeq \lambda$ for $\lambda$ small.]

[6 marks]

To 2 significant digits, how large should $N$ be to achieve an absolute error of approximately $1.8 \times 10^{-11}$? [3 marks]