Computation Theory

Define the primitive recursive and partial ($\mu-$) recursive functions. [6 marks]

Suppose you are given a Turing machine with state set $Q$ and $k$-symbol alphabet $S$ whose action is defined by transition functions

\[
q' = f(q, s) \in Q \uplus \{H\} \quad \text{(disjoint union)}
\]
\[
s' = r(q, s) \in S \quad \text{(replacement symbol)}
\]
\[
d' = d(q, s) \in \{L, R, C\} \quad \text{(movement)}
\]

where the head moves to $L$ or $R$ on the tape unless $q' = H$, in which case $d' = C$ and the machine stops.

Extend the action of the machine by an additional state symbol $D$ so that for all $s \in S$,

\[
f(H, s) = f(D, s) = D
\]
\[
r(H, s) = r(D, s) = s
\]
\[
d(H, s) = d(D, s) = C
\]

Show that the action of the Turing machine as extended in this way can be described by a primitive recursive function $T(t, x)$, where $t$ is a step counter and $x$ is a code specifying the initial configuration. [10 marks]

Hence show that computation by any Turing machine may be represented by a partial recursive function. [4 marks]