

1999 Paper 1 Question 8

Discrete Mathematics

Let Ω be a universal set and define a relation between subsets $A, B \subseteq \Omega$ by $A \cong B \Leftrightarrow \exists$ a bijection $f : A \rightarrow B$. Prove carefully that \cong is an equivalence relation. [6 marks]

What does it mean to say that a set is *countable*? [2 marks]

State without proof the Schröder–Bernstein theorem concerning the existence of a bijection between two sets. [2 marks]

Show that the integers and the rational numbers are countable but that the real numbers are uncountable. [6 marks]

An ML program consists of a finite sequence of characters drawn from a finite alphabet. Show that the set of ML programs is countable. [4 marks]