# 1999 Paper 1 Question 7

## Discrete Mathematics

Define Euler's totient function $\phi(n)$. [2 marks]

Prove the Fermat–Euler Theorem that $a^{\phi(n)} \equiv 1 (\text{mod } n)$ for appropriate $a$. [8 marks]

Deduce a theorem of Fermat about $a^{p-1} - 1$ for a prime number $p$. [2 marks]

Given a prime, $p$, with $p \neq 2$ and $p \neq 5$, show that there are infinitely many natural numbers, each of which has 9s as all its digits and which is divisible by $p$. [8 marks]