

COMPUTER SCIENCE TRIPOS Part IA

Tuesday 1 June 1999 1.30 to 4.30

Paper 2

Answer the question in Section A, **one** question from each of Sections B and C, and **two** questions from Section D.

Submit the answers in five **separate** bundles, each with its own cover sheet. On each cover sheet, write the numbers of **all** attempted questions, and circle the number of the question attached.

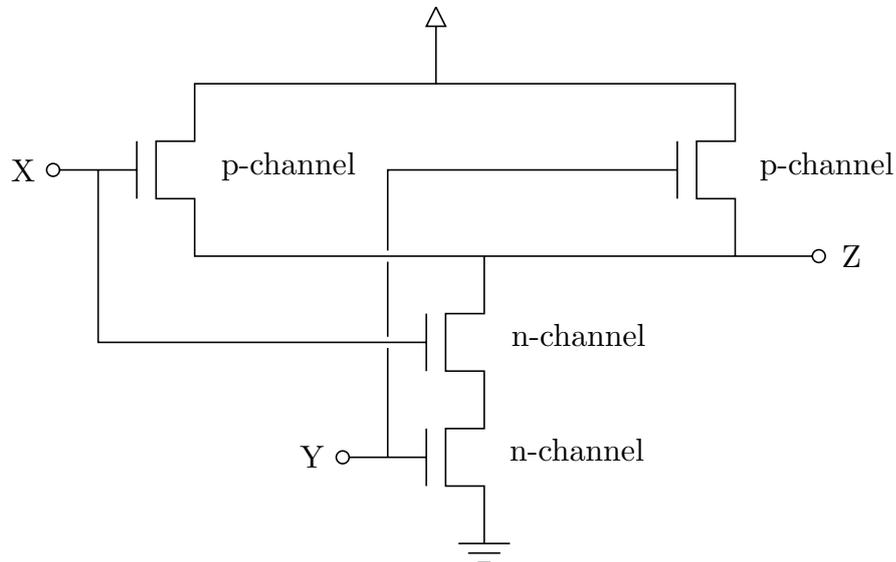
Write on **one** side of the paper only.

SECTION A

1 Twenty-part question (*One mark per part*)

- (a) Consider the ML lists `(fn x => [x,x])(ref 0)` and `map ref [0,0]`. What do they have in common, and how do they differ?
- (b) What does it mean for a function to be *tail-recursive*?
- (c) How can an ML program form a cyclic list? (Your answer need not present any code.)
- (d) Find an integer solution x, y to $284x + 220y = 4$.
- (e) Draw a Venn diagram for three sets, A , B and C , showing the symmetric difference $A \Delta B \Delta C$.
- (f) Define a partial order on set A .
- (g) Give the name of *one* method that must be implemented in a Java stand-alone application, and *two* of the most important methods that might be defined in an applet which just by being defined can alter its behaviour or appearance.
- (h) In Java if `a` is a variable of type `int` what values of `a` will result in `(a & (a-1)) == 0` being `true`?
- (i) What is the range of real numbers which can be represented by a floating point format with 1 sign bit, 8 bias-127 exponent bits, and 23 normalised mantissa bits? You may round the mantissa value to the nearest integer.

- (j) Unix operating systems give each process its own virtual address space, initially containing three segments. What is contained in each of these segments?
- (k) What is this?



- (l) Give a sum of products form for the exclusive or of two variables a and b .
- (m) Give at least one good reason why professionals carry a greater ethical burden in the activities of their profession than ordinary members of the public.
- (n) Write down a short expression or command which is a syntactically correct piece of program, but which would normally be faulted by a type-checker. Justify your answer in a few words.
- (o) Given events A and B , demonstrate that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
- (p) Given the axioms of Probability, prove the Empty Set Theorem: $P(\emptyset) = 0$.
- (q) An inertial navigation set has a mean time to failure of 1000 hours. If an aircraft is fitted with two of them to increase reliability, is the mean time to failure of both systems which is experienced in practice likely to be several thousand hours or about a million hours? Give a brief justification for your answer.
- (r) Outline the key principles of loop design, considering the simple case of a `while` loop.
- (s) What are the differences, if any, between the languages determined by the three regular expressions \emptyset^* , $\emptyset(\emptyset^*)$ and $(\emptyset^*)^*$?
- (t) For the main memory of a computer, why is volatile technology more often used than non-volatile technology?

SECTION B**2 Digital Electronics**

An *arbiter* is a functional unit that allows requesters to lock resources while they are in use.

- (a) Describe the inputs and outputs of an arbiter for a single resource which has two requesters. [5 marks]
- (b) Describe *two* policies that the arbiter might implement when both requesters request the resource simultaneously. One of these should require some internal state. [5 marks]
- (c) Give a state diagram for a policy which requires internal state assuming synchronous operation. [5 marks]
- (d) Choose *one* of the policies in (b) and derive equations for outputs and next state control using any type of storage element you choose. [5 marks]

3 Digital Electronics

In an edge triggered flip flop, explain what is meant by

- (a) hold time
- (b) setup time
- (c) delay from clock edge to output [3 marks]

What relation should hold between two of these quantities to provide sensible behaviour and why? [5 marks]

Comment on the possibility and desirability of negative hold times. [2 marks]

Each of the following boolean functions is a function of the four variables w, x, y, z . The functions are not totally specified and may take on any value for certain inputs. f_n indicates where the function must be true, while g_n is true where the value of the function is not constrained. Draw maps and provide a minimum sum of products form for each of the functions.

$$(a) \quad f_1 = \bar{w}\bar{x}\bar{y}z + \bar{w}x\bar{y}z + \bar{w}xy\bar{z} + wx\bar{y}\bar{z} + wx\bar{y}z + wxy\bar{z}$$

$$g_1 = \bar{w}\bar{x}y\bar{z} + \bar{w}x\bar{y}\bar{z}$$

$$(b) \quad f_2 = \bar{w}\bar{x}\bar{y}\bar{z} + \bar{w}\bar{x}\bar{y}z + \bar{w}x\bar{y}\bar{z} + \bar{w}x\bar{y}z + \bar{w}xy\bar{z} + \bar{w}xyz + w\bar{x}\bar{y}\bar{z} + w\bar{x}y\bar{z} + wxyz$$

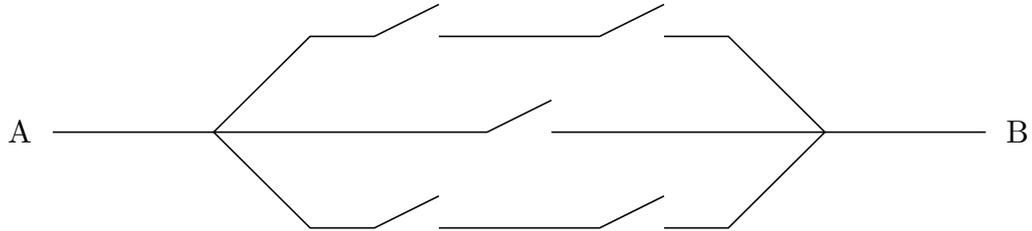
$$g_2 = w\bar{x}y\bar{z} + wxy\bar{z}$$

[5 marks]

Let $f(x_0, x_1, \dots, x_{n-1})$ be equal to 1 if and only if exactly k of the variables have the value 1. How many prime implicants does this function have? [5 marks]

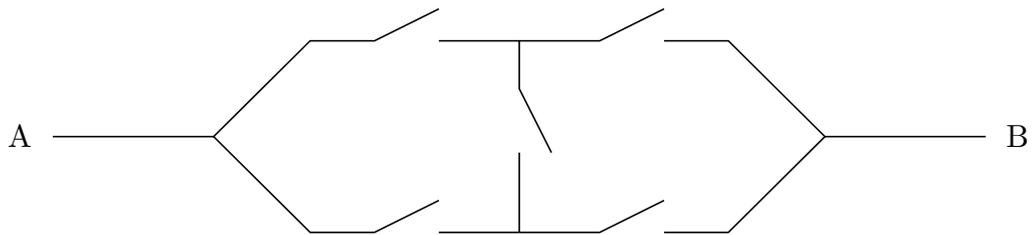
SECTION C

4 Probability



Each of the five switches in the above circuit is closed with probability p , independently of all the other switches. What is the probability of there being a connection from A to B? Express the answer as a polynomial in p . [7 marks]

The circuit is then modified to the version shown below, the five switches still being independent. What is the probability of there being a connection from A to B now? Again express the answer as a polynomial in p . [13 marks]



5 Probability

A solitaire game is played with two coins. One coin is fair, so that $P(\text{heads}) = \frac{1}{2}$, and the other is biased so that $P(\text{heads}) = p$.

At the first turn the player tosses the fair coin. At all subsequent turns the biased coin is used if the toss at the previous turn resulted in heads and the fair coin is used if the toss at the previous turn resulted in tails.

Suppose u_n is the probability of obtaining heads at turn n . Show that, for $n > 1$:

$$2u_n + (1 - 2p)u_{n-1} = 1 \quad [4 \text{ marks}]$$

Demonstrate that this difference equation also holds for the case $n = 1$ provided u_0 is suitably defined. [2 marks]

Solve the difference equation, expressing u_n in terms of n and p . [14 marks]

SECTION D**6 Professional Practice and Ethics**

What is the nature of privacy and how do EU guidelines and/or British legislation serve to protect privacy? [20 marks]

7 Regular Languages and Finite Automata

Suppose that L is a language over the alphabet $\{0, 1\}$. Let L' consist of all strings u' over $\{0, 1\}$ with the property that there is some string $u \in L$ with the same length as u' and differing from u' in at most one position in the string. Show that if L is regular, then so is L' . [Hint: if Q is the set of states of some finite automaton accepting L , construct a non-deterministic automaton accepting L' with states $Q \times \{0, 1\}$, where the second component counts how many differences have been seen so far.] [10 marks]

If a deterministic finite automaton M accepts any string at all, it accepts one whose length is less than the number of states in M . Explain why. [5 marks]

State Kleene's theorem about regular expressions and deterministic finite automata. [2 marks]

Describe how to decide for any given regular expression whether or not there is a string that matches it. [3 marks]

8 Software Engineering I

State the advantages of the waterfall model. [10 marks]

State the disadvantages of the waterfall model. [4 marks]

What is the main criterion for deciding whether or not to use the waterfall model in a software development project? [3 marks]

Explain whether there would be any difference for a hardware development project. [3 marks]

9 Software Engineering II

A Web browser is a complicated program. It must deal with many types of data (images, sound, etc.), support various network services and handle the many constructs of HTML (the language in which Web pages are written). Your manager asks you to lead a small group of programmers in implementing a Web browser. Describe top-down refinement; is it appropriate for your task? [6 marks]

Your manager further states that it is essential that your browser should almost never crash. How would you go about meeting this requirement? [6 marks]

Consider the following two ML functions:

```
fun sumfiv [] = 0
  | sumfiv (x::xs) = 5*x + sumfiv xs;

fun summing z [] = z
  | summing z (x::xs) = summing (z + x) xs;
```

Use structural induction to prove that `sumfiv xs` can be replaced by `5 * summing 0 xs`. [8 marks]

10 Structured Hardware Design

A design is required for an electronic system to collect votes from the audience in a television game show. Each of up to 1000 seats in the audience will have an *audience station* and there will be a *master station* for the floor manager. Output from the system will be video and this will feed monitor displays in the studio and be included in the broadcast programme.

- (a) Sketch a block diagram of the overall system, showing clearly the various units and the signals which make up the cables which run between them. [5 marks]
- (b) Specify the controls and indicators on the front panel of the two types of station. Give a pair of state diagrams of the system, one as seen by the audience and the other as seen by the floor manager. [5 marks]
- (c) Given that a profitable return on the sale of three complete systems is required, describe the components and technology that should be used in the units. [5 marks]
- (d) Discuss the advantages and disadvantages of including a PC inside the floor manager's station, in terms of cost, ease of use and whether a custom or standard operating system should be used. [5 marks]