1998 Paper 9 Question 10

Denotational Semantics

State, with justification, whether each of the following statements is true or false.

(a) The set of natural numbers, $\mathbb{N} = \{0, 1, 2, \ldots\}$, equipped with the usual less-than-or-equal relation, $\leqslant$, is a domain. [3 marks]

(b) The set of all subsets of $\mathbb{N}$, equipped with the relation of subset inclusion, is a domain. [4 marks]

(c) For any domain D and element $d \in D$ with $d \neq \bot$

$$f_d(x) = \begin{cases} \top & \text{if } d \sqsubseteq x, \\ \bot & \text{otherwise} \end{cases}$$

defines a strict continuous function f_d from D to the flat domain $\{\top\}_\bot$. [4 marks]

(d) For any domain D and element $d \in D$ with $d \neq \bot$

$$g_d(x) = \begin{cases} \bot & \text{if } x \sqsubseteq d, \\ \top & \text{otherwise} \end{cases}$$

defines a strict continuous function g_d from D to $\{\top\}_\bot$. [4 marks]

(e) For any continuous functions $h : D \to E$ and $k : E \to D$ between domains D and E, $\mathit{fix}\,(k \circ h) \sqsubseteq k\,(\mathit{fix}\,(h \circ k))$. [5 marks]