Denotational Semantics

State, with justification, whether each of the following statements is true or false.

(a) The set of natural numbers, \( \mathbb{N} = \{0,1,2,\ldots\} \), equipped with the usual less-than-or-equal relation, \( \leq \), is a domain. \( \text{[3 marks]} \)

(b) The set of all subsets of \( \mathbb{N} \), equipped with the relation of subset inclusion, is a domain. \( \text{[4 marks]} \)

(c) For any domain \( D \) and element \( d \in D \) with \( d \neq \bot \)

\[
f_d(x) = \begin{cases} \top & \text{if } d \sqsubseteq x, \\ \bot & \text{otherwise} \end{cases}
\]

defines a strict continuous function \( f_d \) from \( D \) to the flat domain \( \{\top\}_\bot \). \( \text{[4 marks]} \)

(d) For any domain \( D \) and element \( d \in D \) with \( d \neq \bot \)

\[
g_d(x) = \begin{cases} \bot & \text{if } x \sqsubseteq d, \\ \top & \text{otherwise} \end{cases}
\]

defines a strict continuous function \( g_d \) from \( D \) to \( \{\top\}_\bot \). \( \text{[4 marks]} \)

(e) For any continuous functions \( h : D \to E \) and \( k : E \to D \) between domains \( D \) and \( E \), \( \text{fix} (k \circ h) \sqsubseteq k (\text{fix} (h \circ k)) \). \( \text{[5 marks]} \)