1998 Paper 8 Question 15

Communicating Automata and Pi Calculus

A process is *deterministic* if, for any state P and any action a, there is at most one transition $P \xrightarrow{a} P'$. If $s = a_1 \cdots a_n$ is a sequence of actions, then $P \xrightarrow{s} P'$ means $P \xrightarrow{a_1} \cdots \xrightarrow{a_n} P'$.

By drawing a transition graph, define a deterministic sequential process Q which uses the four actions $\mathsf{in}_1, \mathsf{out}_1, \mathsf{in}_2, \mathsf{out}_2$ such that, for every action sequence s, $Q \xrightarrow{s} Q'$ for some Q' if and only if s satisfies the four conditions

(a) $0 \leqslant \#_s\mathsf{in}_1 - \#_s\mathsf{out}_1 \leqslant 1$

(b) $0 \leqslant \#_s\mathsf{in}_2 - \#_s\mathsf{out}_2 \leqslant 1$

(c) $0 \leqslant \#_s\mathsf{in}_1 - \#_s\mathsf{in}_2 \leqslant 1$

(d) $0 \leqslant \#_s\mathsf{out}_1 - \#_s\mathsf{out}_2 \leqslant 1$

where $\#_s a$ is defined to be the number of times a occurs in s. For example the sequence $s = \mathsf{in}_1\,\mathsf{in}_2\,\mathsf{out}_1$ satisfies the conditions, but $s = \mathsf{in}_1\,\mathsf{in}_2\,\mathsf{out}_2$ violates (d). Also write down the defining equations of Q.
[Hint: Q need have no more than eight states.] [8 marks]

Define *weak bisimulation* and *weak equivalence* ($\approx$), and explain their advantages over the strong versions. [4 marks]

With the help of one or more extra actions $\vec{a}$, define two communicating processes R_1 and R_2 with no more than four states each, such that

$$Q \approx \mathsf{new}\,\vec{a}(R_1 \mid R_2)$$

and verify this weak equivalence. [8 marks]